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A NEW TREATMENT SOLUTION OF INTERVAL NONLINEAR PROGRAMMING PROBLEMS: A CASE STUDY OF GREEN FUEL PRODUCTION

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Abstract

Green fuel is growing in popularity in recent years. Bio-reactive waste converted to green fuel through anaerobic digestion technology. The performance of biogas unit has been optimized and formulated as interval programming problems as function of inlet feed rate, retention time fermentation temperature and pH. A new treatment for solving the interval nonlinear programming problem (INPP) is discussed. All the intervals in the INPP are replaced by new variables. This the modified nonlinear programming problem (MIPP). We presented three hybrid evolutionary algorithms (EAs) which are chaotic genetic algorithm (CGA), chaotic particle swarm optimization (CPSO) and chaotic firefly algorithm (CFA) to solve MIPP. The Karush–Kuhn–Tucker (KKT) conditions for MIPP are gotten. These equations are solved as algebraic equations. Its solutions may be represented as a function of new variables to get the stability set of first kind. The staring points in EAs is gotten by the Newton method. Finally, the comparison between the stability set of first kind, CGA, CPSO and CFA are presented with discussion. An empirical optimization model of biogas production has been constructed with accuracy of 90%.

Keywords: Interval Nonlinear Programming Problem, KKT Conditions, Stability Set Of First Kind, Genetic Algorithm, Particle Swarm Optimization, Firefly Algorithm, Green Fuel Production, Biogas Production Process

1. INTRODUCTION

Linear and nonlinear programming problems are usually used to model and solve real application [1,2,3]. Interval nonlinear programming problem is very interesting area [4-14]. With undetermined coefficients in both nonlinear objective function and constraints, the nonlinear interval number programming problem is resolved by Jiang et al. [13]. Planning of waste management activities is one of the real-world applications that modeled and solved by INPP [15]. The key mathematical justifications for interval analysis are readily available in [16]. Numerous researchers and authors use a variety of approaches to handle interval nonlinear programming issues [17–19], but they all aim to find the best solution for a given set of circumstances. A numerical approach to solving an interval nonlinear programming issue was described by Liu and Wang [18]. To solve interval quadratic programming, Li and Tian [19] expand Liu and Wang's [18] method, which takes less time to compute than Liu and Wang's method.

The most common method to solve INPP is divided into two problems. For instance, Hladk separated the issue in [10, 11] into subclasses that could be reduced to simple issues that required convex programming. While the second problem finds the best solution for the higher goal function in the smallest feasible zone, the first problem finds the best solution for the lower objective function in the greatest feasible region. Therefore, the value of the interval problem's solution is between the values of the lower and upper objective functions. In many circumstances, this method is exceedingly challenging, which makes it challenging to determine the ideal value of the problem's objective function.

Over the last two decades, there was a focusing on evolutionary algorithms (EAs) due to the drawbacks of the numerical methods, such as the need to derivative the search domain, the probability of dropping into local optima, and they cannot solve all nonlinear problems [20]. EAs have attracted great interest in engineering, and industry [21]. Different natural swarm systems based EAs have been effectively used in practical applications. There are numerous EAs, including the genetic algorithm (GA) [22], the particle swarm optimization (PSO) [23], the fruit fly algorithm (FFA) [24], the manta-ray foraging optimization algorithm (MRFOA) [25], the grasshopper optimization algorithm (GOA) [26], the sine cosine algorithm (SCA) [27] the firefly algorithm (FA) [28], etc.

The KKT optimality conditions for interval-valued objective functions and real-valued constraint functions were discussed in [29-32]. The mathematical basics of duality theorems in interval-valued optimization problems are investigated in [8,9,10,14].

Many papers studied the parametric nonlinear programming problems [33-35] since Osman derived the stability set of first kind and the stability set of second kind [34, 35]. We use the first kind set for solving interval nonlinear programming. As seen in pervious papers [34,35], the stability set of first kind is a set of parameters which produces the same optimal solution. The relationship between optimal solution of INPP and the change of value selected from intervals. This important relation from the point of view to the decision maker. This property is used to solve INPP.

As result of increasing energy demand versus probable fossil fuel exhaustion and change in climate. To reduce greenhouse gas emissions and other environmental repercussions from the energy sector, many nations seek out renewable and alternative energy sources. [36]. The annual garbage creation rate is rising due to the expanding urban population as shown in figure 1.

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Fig 1: Human waste.

Consequently, a significant disposal of wastes is very important step for reduction of waste propagation [37]. Waste-to-energy trend has been carried out by different technologies as illustrated in figure 2.

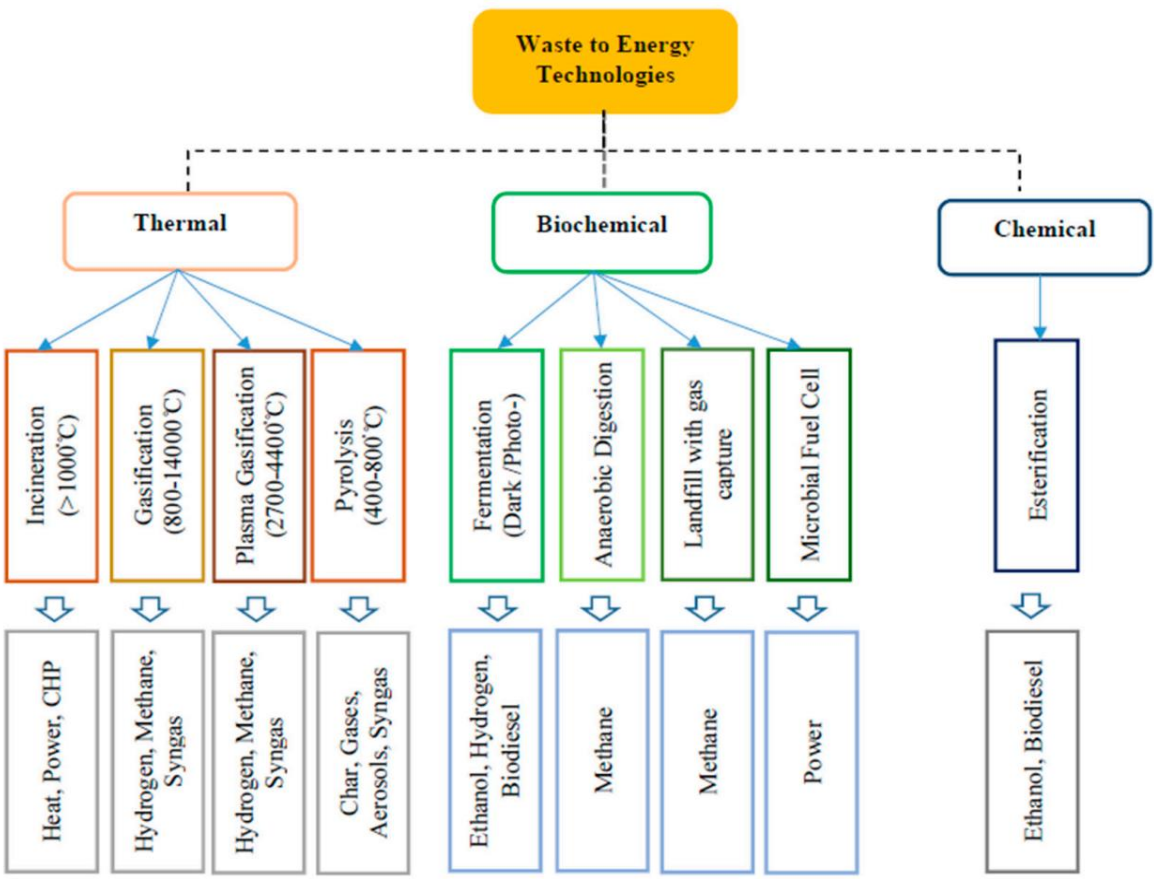

Fig 2: Common waste-to-energy innovations within [38].

Waste to energy has been considered as a green energy source that can be used to meet high demand for true fossil-free energy [39], as well as to lessen the outflow of greenhouse gas and the influence of environmental change. Today, it has modified into a type of green energy use that can assist both the environment and the global economy [40].

In this paper, a parametric approach is suggested to solve INPP. INPP are converting to MIPP by replacing all intervals to new variables. Three hybrid EAs are proposed which are CGA, CPSO and CFA to solve MIPP and its dual problem. KKT conditions of MIPP are derived. The KKT equations are solved and can be expressed as a function in the new variables to get the stability set of first kind. So, we can get the optimal solution easily at any value of intervals. The dual of MIPP is discussed. We use the solution of dual MIPP to improve the behavior and as a stopping criterion for EAs.

The following are the amin contributions of the current paper:

1. Three hybrid EAs, which are CGA, CPSO and CFA, is presented to solve MIPP and evaluated.
2. A comparative study of the optimization results using the hybrid EAs, and the stability set of first kind is presented of 2 benchmark test problems.
3. The results proved that the hybrid algorithms have a rapid convergence to the optimal solution as the CPU time is very small compared to the numerical methods that need high effort of computation to solve this type of problem. But by using the stability set of first kind, we can detect the optimal solution easily respect to any perturbations on intervals.
4. We formulae and optimize the performance of biogas unit as interval programming problems as function of inlet feed rate, retention time fermentation temperature and pH.

The rest of the paper is organized as follows: In section 2, a briefly introduction of INPP and its dual problem. Section 3 presents biogas production process while section 4 shows the proposed approach. In section 5, test problems and a real application are discussed. Finally, the conclusion is given in section 6.

2. INTERVAL NONLINEAR PROGRAMMING PROBLEM (INPP)

INPP is defined as [4-14]:

$$\begin{aligned} &\min \quad \sum_{i=1}^{k} y_i^f f_i(x), \\ &\text{subject to:} \sum_{i=1}^{l} y_{ij}^c g_{ij}(x) \le y_j^R \,, j = 1,2,\dots,m, \end{aligned} \tag{1}$$

where $\sum_{i=1}^{k} y_i^f f_i(x)$ is interval-valued function, $y_i^f = \left[y_i^{fL}, y_i^{fU}\right] \forall i, f_i(x)$ is a nonlinear function and $g_{ij}(x), i = 1,2,\dots,l, j = 1,2,\dots,m$ are linear/nonlinear functions, continuous and differentiable on R^n, and $y_{ij}^c = [y_{ij}^{cL}, y_{ij}^{cU}]$, $y_j^R = [y_j^{RL}, y_j^{RU}]$, $i = 1,2,\dots,l, j = 1,2,\dots,m$. The feasible region is nonempty and fixed.

The problem (1) can be divided on two sub problem. The first problem can be written as follows:

$$\begin{aligned} &\min \quad \sum_{i=1}^{k} y_i^{fL} f_i(x), \\ &\text{subject to: } \sum_{i=1}^{l} y_{ij}^{cL} g_{ij}(x) \le y_j^{RU}, j = 1,2,\dots,m; \end{aligned} \qquad (2)$$

Assume that the optimal solution of problem (2) is $\bar{x}$ and $\overline{f^L}$ is its optimal value.

The second problem can be written as follows:

$$\begin{aligned} &\min \quad \sum_{i=1}^{k} y_i^{fU} f_i(x), \\ &\text{subject to:} \sum_{i=1}^{l} y_{ij}^{cU} g_{ij}(x) \le y_j^{RL}, j = 1,2,\dots,m, \end{aligned} \qquad (3)$$

Assume that $\bar{\bar{x}}$ is the optimal solution of problem (3) and $\overline{f^U}$ is its optimal value. If $\bar{x} = \bar{\bar{x}}$, then it can be called the optimal solution of problem (1) and $\min_{x \in X} f(x) \in [\overline{f^L}, \overline{f^U}]$. The goal of studying duality is to ensure that the optimal solution is obtained. The duality form may be simpler or less difficult than the primal one, for more details read [1]. We use this property as a new stopping criterion to our algorithms. In papers [4,7,8,9,10,11] the authors discussed the duality theorems for different types of interval programming problems. All these papers proved a different form of duality, all these theorems are difficult. So, we use the Lagrangian method which is a powerful constructive dual search method. We must replace all intervals by new variables and converting INPP to MIPP. The dual problem of INPP is defined as following:

$$\begin{aligned} &\max \quad \theta(\mu, a, b), \\ &\text{subject to: } \mu \ge 0, \end{aligned} \qquad (4)$$

where $\theta(\mu, a, b) = \inf\{\sum_{i=1}^{k} a_i f_i(x) + \sum_{i=1}^{m} \mu_i \sum_{i=1}^{l} (b_{ij} g_{ij}(x) - b_j) \mid x \in R^n\}$, $a_i = [y_i^{fL}, y_i^{fU}]$, $b_{ij} = [y_{ij}^{cL}, y_{ij}^{cU}]$, $b_j = [y_j^{RL}, y_j^{RU}]$.

3. BIOGAS PRODUCTION PROCESS

Biogas production process is defined as the conversion of complex biomass have into methane by the action of anaerobic microbes [41]. In biogas process cultivate squanders or vitality crops are treated anaerobically in a fermenter. These plants are mixed with maize silage or biodegradable sewage slime. Amid the method, the micro-organisms change biomass squander into biogas (primarily methane and carbon dioxide) and residuals. Higher amounts of biogas can be generated when the wastewater is co-digested with industrial residuals (organic industries). For case, whereas blending 90% of wastewater from brew manufacturing plant with 10% dairy animals' whey, the generation of biogas was expanded by 2.5 times more than that delivered by wastewater from the brewery as it were [42]. A simplified biogas production digestor is displayed in figure 3.

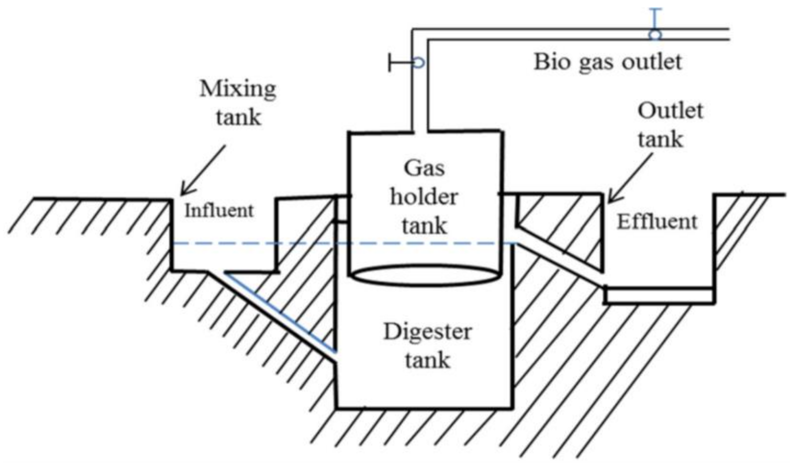

Fig 3: Simple biogas plant schematic [43]

The biogas could be a maintainable source of vitality because, it is completely vitality self-sufficient, autonomous of any fossil fuel, renewable, carbon impartial and decreases the emanation of nursery gasses (GSGs) to the environment. Bio-substrates as it were emanating the carbon dioxide, they have amassed amid their life cycle and which they would have transmitted moreover without the enthusiastic utilization. In general, power delivered from biogas produces much less carbon dioxide than ordinary vitality and hence will be accommodating in diminishing nursery gas emanation [44]. The resultant gas consists of CH_4 (65%) and CO_2 (35%) and some other gases (H_2S), N_2, etc. [45]. The whole process has been summarized in figure 4.

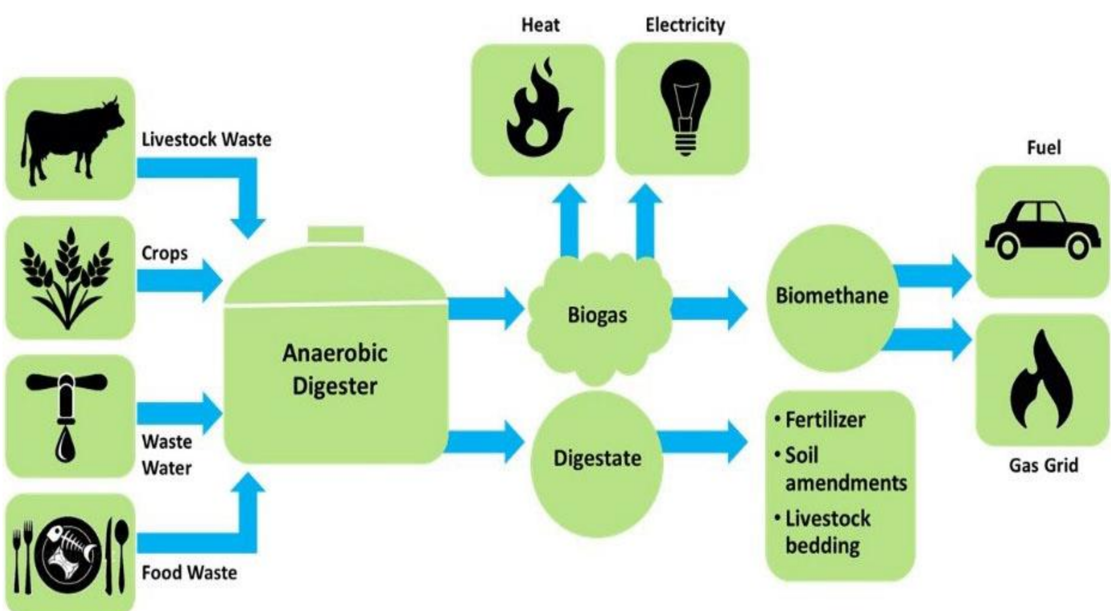

Fig 4: Anaerobic digestion process.

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3.1 Biochemical mechanisms of biogas process

Human and animal wastes are digested anaerobically to produce biogas through a variety of microbial communities, Bacteria that oversee hydrolyzing high molecular weight organic compounds start the AD chain. The created mono- and oligomers are at that point encourage debased to unstable greasy acids (VFAs), taken after by acetic acid, CO_2 and H_2, which are all considered. The last phase (methanogenesis) is carried out by converting acetic acid or CO2/H2 into methane [46].

3.2 Chemical biogas production mechanism

The biogas production process could be generalized in Eq. (5) which in turn subdivided into several equations according to bio digestion steps.

$$biomass + organism \rightarrow \mathrm{CH4} \qquad (5)$$

Biogas production steps [47], there are four steps involved in biogas production process: Hydrolysis, Acidogenesis, Acetogenisis and Methanogenesis. Hydrolysis step is known as the liquefaction. As the insoluble complexes are transformed into a simple soluble compound via bacterial fermentation (conversion of cellulose to sugar). Hydrolysis step is the limiting steps and expressed by Eq. (6).

$$\mathrm{C_6H_{10}O_4 + 2H_20 \rightarrow C_6H_{12}O_6 + 2H_2} \qquad (6)$$

After hydrolysis step, simple molecules are converted to Acidogenesis volatile fatty acids Eqs. (7), (8) and (9).

$$\mathrm{C_6H_{12}O_6 \Leftrightarrow 2\,CH_3CH_2OH + 2CO_2} \qquad (7)$$

$$\mathrm{C_6H_{12}O_6 + 2H_2 \Leftrightarrow 2CH_3CH_2COOH + 2H_2O} \qquad (8)$$

$$\mathrm{C_6H_{12}O_6 + 2H_20 \rightarrow CH_3COOH} \qquad (9)$$

Consequently, in acetogenesis the product from the second process is converted into the simple organic acid (acetic acid, propionic acid, and ethanol), carbon dioxide and hydrogen, Eqs. (10), (11), and (12).

$$\mathrm{CH_3CH_2COO^{\mp}\ 3H_20 \Leftrightarrow CH_3COO^{\mp} H^{+} + HCO_3^{-} + 3H_2} \qquad (_{10})$$

$$\mathrm{C_6H_{12}O_6 + 2H_{2O} \Leftrightarrow 2CH_3COOH + 4H_2 + 2CO_2} \qquad (11)$$

$$\mathrm{CH_3CH_2OH + 2H_20 \rightarrow CH_3COO^{-} + 2H_2 + H^{+}} \qquad (12)$$

Finally, Methanogenesis is responsible to methane production by methanogens bacteria. Methane is produced either by are through reduction of carbon dioxide with the hydrogen cleavage of acetic acid to carbon dioxide and methane or, Eqs. (13), (14) and (15).

$$C\mathrm{O_2 + 4H_2 \rightarrow CH_4 + 2H_2O} \qquad (13)$$

$$\mathrm{2C_2H_5OH + C0_2 \rightarrow CH_4 + 2CH_3COOH} \qquad (14)$$

$$\mathrm{CH_3COOH + 2H_20 \rightarrow CH_4 + CO_2} \qquad (15)$$

The process of decomposing organic matter anaerobically involves several different degradation processes carried out by various microbial consortia thus [43], There are several factors affection biogas production process such as, nature of feedstock and substrate, digester design, inoculant, pH, temperature, loading rate and hydraulic retention time. All these factors could be optimized to obtain the maximum gaseous yield at minimum cost. The produced methane is utilized as fuel and the removed fermentation residuals(digestate) are utilized as organic fertilizer. Briefly biogas production process is regarded as a promising step towards waste to energy. Waste to energy performance is illustrated by the following case study, 500 heads of dairy calves whose fertilizer is bolstered into a plug-flow anaerobic digester produces 635 ft^3/d of biogas which in turn generated 81,000 Btu/d.

4. THE PROPOSED APPROACH

In order to solve INPP, our method first converts INPP to MIPP by substituting all intervals with new variables.

$$\begin{aligned} &\min \quad \sum_{i=1}^{k} a_i f_i(x), \\ &\text{subject to:} \sum_{i=1}^{l} b_{ij} g_{ij}(x) \le b_j\,, j = 1,2,\dots,m, \end{aligned} \qquad (16)$$

Definition 1: The solvability set of problem (16) is defined as:

$$SSP = \{(a,b) \in A \times B \,|\, SO(a,b) \neq \emptyset\}; \qquad (17)$$

where the set of all optimal points of problem (16) is $SO(a,b)$.

$$SO(a,b) = \left\{ \bar{x} \in R^n \middle| f(\bar{x}) = \min_{x \in \{\sum_{i=1}^{l} b_{ij} g_{ij}(x) \le b_j, j=1,2,\dots,m\}} \sum_{i=1}^{k} a_i f_i(x) \right\} \qquad (18)$$

Definition 2: The stability set of the first kind of problem (16) is defined as:

$$SF(\bar{x}) = \left\{ (a,b) \in SP \middle| f(\bar{x}) = \min_{x \in \{\sum_{i=1}^{l} b_{ij} g_{ij}(x) \le b_j, j=1,2,\dots,m\}} \sum_{i=1}^{k} a_i f_i(x) \right\}; \qquad (19)$$

where $\bar{x}$ is the optimal solution of problem (5).

we formulate KKT conditions of MIPP as:

$$\begin{aligned} &\sum_{i=1}^{k} \nabla a_i f_i\,(x) + \nabla \sum_{j=1}^{m} \mu_j \sum_{i=1}^{l} \left(b_{ij} g_{ij}(x) - b_j\right) = 0. \\ &\mu_j \sum_{i=1}^{l} \left(b_{ij} g_{ij}(x) - b_j\right) = 0, j = 1,2,\dots,m. \\ &\sum_{i=1}^{l} b_{ij} g_{ij}(x) \le b_j\,, j = 1,2,\dots,m. \\ &\mu_j \ge 0, j = 1,2,\dots.,m. \end{aligned} \qquad (20)$$

On the other hand, three algorithms which are CGA, CPSO and CFA to solve MIPP, are presented; where the performance of GA, FA and PSO are improved by using chaos

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theory (CT) [48]. The convergence and diversity of the solutions are improved by the chaotic sequence's high randomness. Due to the use of chaotic maps, CT is regarded as having irregular behavior in nonlinear systems. It was applied to enhance the quality of the solutions produced by algorithms like hybrid chaos-PSO [49], chaotic enhanced GA [50], Chaotic search-based equilibrium optimizer (EO) [51], chaotic search-based salp swarm algorithm (SSA) [52], and chaotic artificial neural networks (ANN) [53].

The proposed approach steps can be summarized as:

1. KKT conditions of MIPP are derived. To solve a system of nonlinear equations with interval coefficients, utilize the theorems stated in [54]. Let MIPP have a continuous function $F: A_0 \subseteq \mathbb{I}R^n \rightarrow \mathbb{I}R^n$ which has a zero u^* in a given subset A of A_0, i.e. a vector $u^* \epsilon A \subseteq A_0$ exists such that $F(u^*) = 0$. Where $\mathbb{I}R^n$ is the set of real intervals n vectors. The following symbols are used, R^n is the set of real n vectors, $R^{n\times n}$ is the set of $n \times n$ matrices, $\tilde{u}$ is an element of an interval vector u, B^H is a hull inverse of the $n \times n$ interval matrix B, $int(A)$ is an interior of the $m \times n$ interval matrix A, $\mathbb{I}A$ is the set $\{u \in \mathbb{I}R^n | u \subseteq A\}$, $int(u) \equiv]\underline{u}, \overline{u}[$ is the interior of an interval u and $\mathrm{vol}(u)$ is a volume-reducing property of the Newton iteration.
2. These are algebraic equations that can be solved. To obtain the stability set of first order, the solutions can be written as a function in the additional variables. If the optimal solution at specific interval coefficient values is needed, it is highly beneficial.
3. To determine the boundaries of the variables, the Newton approach is employed, which shortens the calculation time for EAs.
4. Newton method is handled and obtaining the first-kind stability set by using the Mathematica program.
5. Rather than employing the entire space of the variables, the solutions of the second technique are employed as the first stage of CGA, CPSO, and CFA to enhance their ability to identify the solution quickly.
6. To solve MIPP and its dual, CGA, CPSO, and CFA are use
7. As a stopping criterion for EAs, the dual MIPP solution is used. The new stopping criteria is suggested here is using the dual problem solution. The objective function values that result from solving MIPP are compared with its dual form. We can say that our problem is solved globally optimally if their values are same. If there are discrepancies between them, it is indicated by the symbol δ. We resolve the problem and its dual once more until there is no longer any difference ε between them; $\varepsilon = \frac{\delta}{\text{The optimal value of problem}}$. This solution is local optimal solution. The suggested strategy works for both convex and nonconvex problems.

4.1. Chaotic genetic algorithm (CGA):

To obtain a global or nearly global optimal solution, GA was first developed in the 1960s [55,56]. The fundamental steps of CGAs are outlined below:

Step 1: Create a population of chromosomes at random that will work as the problem's solutions.

Step 2: Evaluate each chromosome's fitness value in the population.

Step 3: Repeat the steps below to create a new population.

a) Use the selection method to pick two parents from a population.

b) To produce the new offspring, the parents are crossed.

c) New offspring are mutated.

Step 4: Chaotic repairing the new population

Step 5: Use the newly formed population to run the algorithm again.

Step 6: Stop the run if a predetermined halting condition is met; otherwise, go back to step 2.

Figure 5 depicts the flowchart used to demonstrate GA.

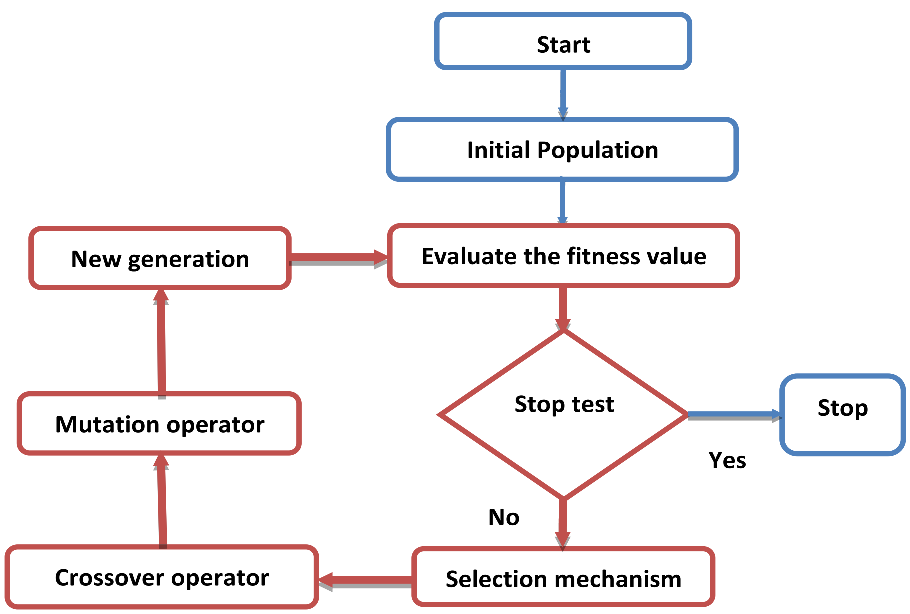

Fig 5: The flowchart of GA

4.2. Chaotic particle swarm optimization algorithm (CPSO):

PSO is a population-based methodology that draws its inspiration from the flocks of birds. PSO has a faster convergence rate and can solve many challenging optimization issues. Additionally, PSO's implementation is simple because it contains only a few parameters [57]. The *i*-th particle is described by a *n*-dimensional vector as $x_i = (x_{i1}, x_{i2}, \dots, x_{in})$, while its velocity is represented as $v_i = (v_{i1}, v_{i2}, \dots, v_{in})$. The best position of the particle in its memory that it visited, denoted as $p_i^{best} = (p_{i1}, p_{i2}, \dots, p_{in})$. $g^{best} = (g_1, g_2, \dots, g_n)$ represents the best location in the swarm. The steps of the CPSO algorithm are described as follows:

Step 1. Initialization

Initialize randomly the positions and velocities of all particles, and set $t = 1$; where t the increment of time.

Step 2. Optimization

(a) Evaluate f_i^t: the value of the objective function.

(b) If $f_i^t \le f_i^{best}$ then $f_i^{best} = f_i^t$ and $p_i^{best} = x_i^t$.

(c) If $f_i^t \le f_g^{best}$ then $f_g^{best} = f_i^t$ and $g_i^{best} = x_i^t$.

(d) Proceed to step 3 if the halting criterion is met.

(e) According to the following equation updated all velocities:

$$v_i^{t+1} = wv_i^t + c_1 \times r_1 \times (p_i^{best} - x_i^t) + c_2 \times r_2 \times (g_i^{best} - x_i^t) \forall\, i - th\ particle\ ; \qquad (21)$$

where w an inertia terms, $c_1 \& c_2$ positive constant, and $r_1 \& r_2$ random numbers belong to (0,1).

(f) According to the following equation updated all positions:

$$x_i^{t+1} = x_i^t + v_i^{t+1} \forall\, i - \text{the particle}. \qquad (22)$$

(g) Repairing the new position x_i^{t+1} chaotically.

(h) *t=t+1*.

(i) Go to Step 2.

Step 3. Termination

Stop the run if a predetermined stopping requirement is met; otherwise, proceed to step 2. Figure 6 depicts the flowchart that represents PSO.

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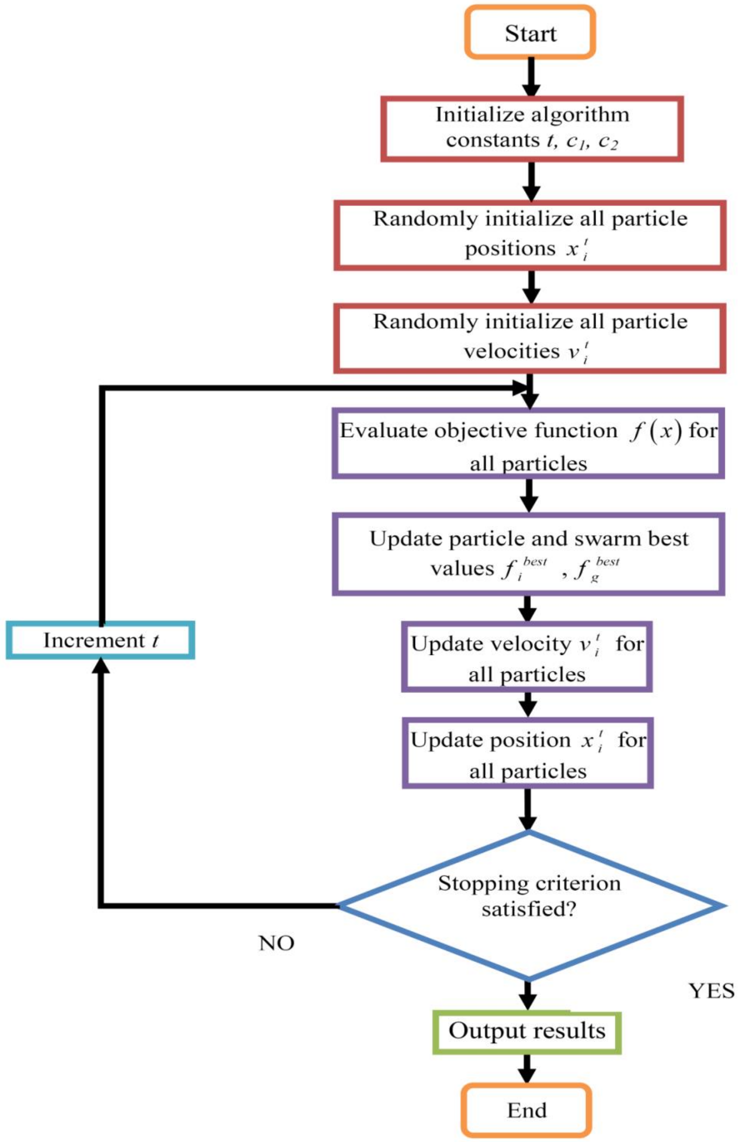

Fig 6: The flowchart of PSO.

4.3. Chaotic Firefly algorithm (CFA):

The FA approach uses evolutionary computation to address optimization issues [58]. The following is a description of the CFA's main steps:

Step 1. Initialization:

The position of the *i*-th firefly in an *n*-dimensional space is indicated as x_i and written as $x_i^t = (x_{i1}, x_{i2}, \dots, x_{in})$. At time $t = 0$, a population of N fireflies are initialized at random.

Step 2. Evaluation:

Each firefly's fitness value is evaluated as $I(x_i^t) = f(x_i^t) \forall i = 1,2,\ldots,N$.

Step 3. Choosing of best solution:

The firefly with the lowest light intensity x_b is the best answer, for minimization problems.

Step 4. Updating firefly positions:

Do the following for each firefly $i = 1,2,\ldots,N$ and for each firefly $j = 1,2,\ldots,N$. According to the following equations, if $I(x_j^t) < I(x_i^t)$, then the *i*-th firefly will be drawn to firefly *j* with updating its position x_i^{t+1}:

$$x_i^{t+1} = x_i^t + \beta_0 e^{-\gamma r_{ij}^2}\left(x_j^t - x_i^t\right) + \alpha_k \varepsilon_k, \quad (23)$$

$$r_{ij=}\left|x_i^t - x_j^t\right| = \sqrt{\sum_{d=1}^{d}\left(x_{id}^t - x_{jd}^t\right)^2}; \quad (24)$$

where r_{ij} is the cartesian distance between the two fireflies *i* and *j,* β_0 is attractiveness at r_{ij} is 0, α_k is a parameter controlling the step size, γ is the light absorption coefficient, and ε_k is a vector drawn from a Gaussian or other distribution. If $I\left(x_j^{t+1}\right) < I(x_i^t)$ then $x_i^{t+1} = x_i^{t+1}$ otherwise $x_i^{t+1} = x_i^t$

Step 4-1. Repairing in a chaotic manner the new position x_i^{t+1}**.**

Step 4-2. If the *i*-th firefly's new position, x_i^{t+1}, is superior to the best solution, x_b, i.e., $I\left(x_j^{t+1}\right) < I(x_b)$. Then, the best solution, x_b, is updated as $x_b = x_i^{K+1}$.

Step 5. Stopping Condition:

Stop the run if a predetermined halting requirement is met; otherwise, proceed to step 4. Figure 7 depicts a flowchart that shows FA.

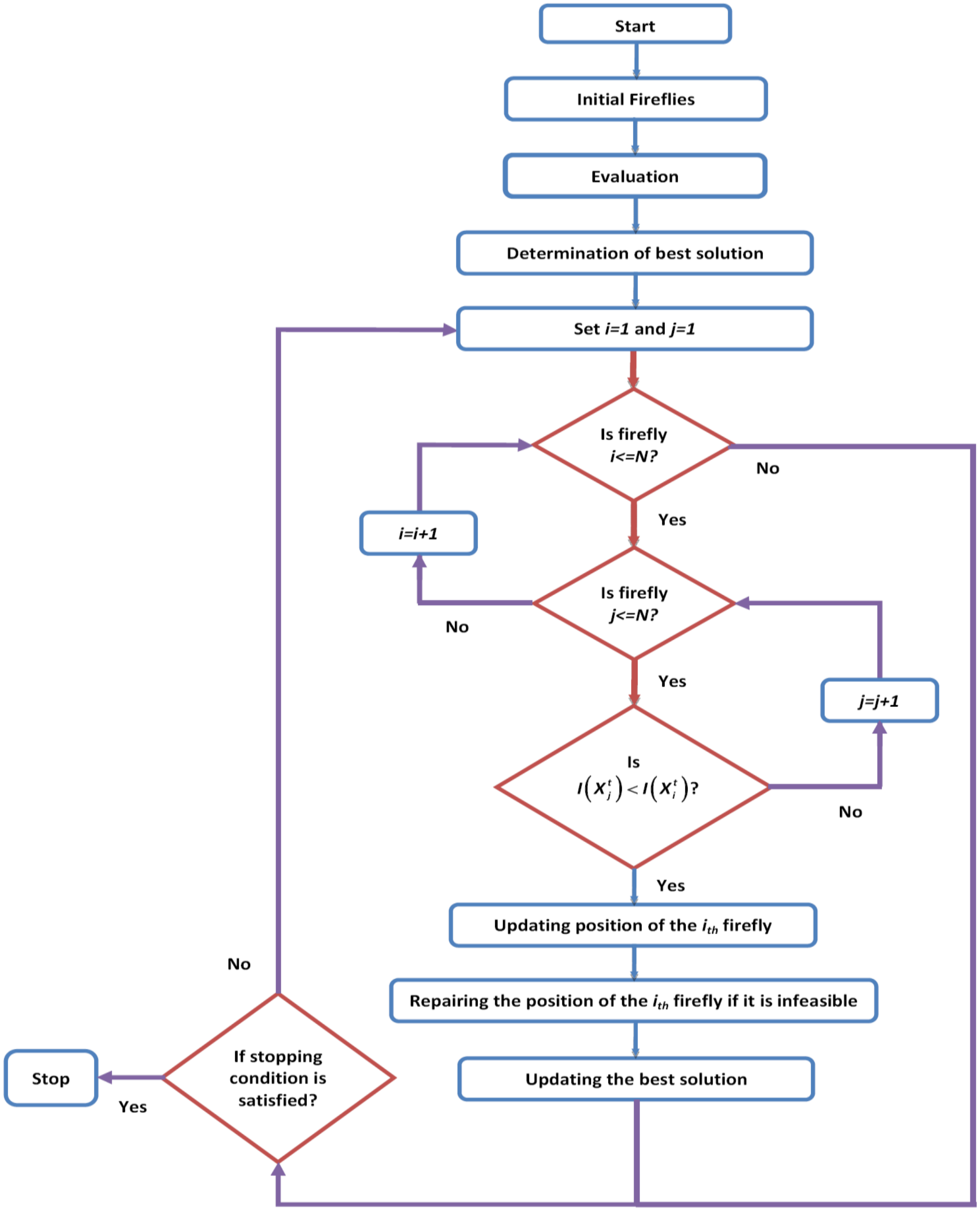

Fig.7: The flowchart of FA.

4.4. Repairing an infeasible solution in a chaotic manner

If the new solution x_i^{t+1} is infeasible, it is fixed in accordance with the subsequent equation:

$$x_i^{t+1} = \vartheta \times x_i^{t+1} + (1 - \vartheta) \times BS \tag{25}$$

If x_i^{t+1} is still infeasible, x_i^{t+1} it is fixed in accordance with the subsequent equation:

$$x_i^{t+1} = \vartheta \times BS + (1-\vartheta) \times x_i^{t+1} \qquad (26)$$

where BS is any the best solution obtained so far and ϑ is generated by the following chaotic map:

$$\vartheta_{m+1} = c\,\vartheta_m(1-\vartheta_m),\ c = 4,\ \ \vartheta_0 \in (0,1)\ \text{ and }\ \vartheta_0 \notin \{0, 0.25, 0.5, 0.75, 1\}; \qquad (27)$$

where m is the is the age of the infeasible solution. In other words, a number is generated by the chaotic map as in Eq. (27). Then the infeasible solution is repaired according to Eqs. (25) and (26) until to be feasible. If the infeasible solution, after m iterations, not be feasible, a new solution is reinitialized instead of this infeasible one.

5. COMPUTATIONAL EXPERIMENTS

The proposed approach is put to the test on two problems picked from the literature to gauge how well it performs. Furthermore, the case study green fuel generation is employed to demonstrate its suitability for handling practical applications. The algorithms are programmed using MATLAB (R2016b) on a computer with a P4 CPU running at 3.00 GHz, 1 GB of RAM, and an i5 processor. Each algorithm has several factors that influence how well it performs. Table 1 is a list of the criteria used to implement CGA, CPSO, and CFA.

Table 1: The proposed approach design parameters

CGA parameters		**CPSO parameters**		**CFA parameters**	
The swarm size	20	**The swarm size**	20	**The swarm size**	20
Number of iteration (T)	100-200	**Number of iteration (T)**	100-200	**Number of iteration (T)**	100-200
Selection operator	Stochastic universal sampling	**Acceleration coefficients**	$c_1 = 2.8$	**Initial attractiveness (β_0)**	1
Crossover operator Crossover rate	Single point 0.8		$c_2 = 1.3$	**The light absorption coefficient (γ)**	1
Mutation operator Mutation rate	Real-value 0.06	**The inertia weight (w)**	0.6	**The step size factor (α)**	0.95
Chaos search repairing iteration (m)	100				
ε	1E-6				

5.1. Problem 1:

The formulation of this problem is as follows [13]:

$$\min\ [2,3]x_1^2 + 2x_2^2 - 2x_1x_2 + [-5,-3]x_1 + [1,2]x_2 \qquad (28)$$

$$\text{subject to: } [1,2]x_1 + x_2 \le [2,4], [2,3]x_1 + [-1,-0.5]x_2 \le [3,4],$$
$$[4,5]x_1 + [-8,-7]x_2 = [1,1.5],$$
$$x_1, x_2 \ge 0.$$

All intervals are replaced by an additional parameter, the problem becomes:

$$\min a_1x_1^2 + 2x_2^2 - 2x_1x_2 + a_2x_1 + a_3x_2$$
$$\text{subject to: } b_1x_1 + x_2 \le b_2, b_3x_1 + b_4x_2 \le b_5, b_6x_1 + b_7x_2 = b_8, \qquad (29)$$
$$x_1, x_2 \ge 0;$$

where $a_1 = [2,3], a_2 = [-5,-3], a_3 = [1,2], b_1 = [1,2], b_2 = [2,4], b_3 = [2,3],$
$b_4 = [-1,-0.5],\ b_5 = [3,4], b_6 = [4,5], b_7 = [-8,-7]$ and $b_8 = [1,1.5].$

The duality version of problem (29) is:

$$\max a_1x_1^2 + 2x_2^2 - 2x_1x_2 + a_2x_1 + a_3x_2 + u_1(b_1x_1 + x_2 - b_2) + u_2(b_3x_1 + b_4x_2 - b_5) + \lambda(b_6x_1 + b_7x_2 - b_8) - u_3x_1 - u_4x_2. \qquad (30)$$
$$u_1 \ge 0, u_2 \ge 0, u_3 \ge 0, u_4 \ge 0.$$

The KKTCs of problem (30) is:

$$2a_1x_1 - 2x_2 + a_2 + u_1b_1 + u_2b_3 + \lambda b_6 - u_3 = 0,$$
$$4x_2 - 2x_1 + a_3 + u_1 + u_2b_4 + \lambda b_7 - u_4 = 0,$$
$$u_1(b_1x_1 + x_2 - b_2) = 0,\ u_2(b_3x_1 + b_4x_2 - b_5) = 0, \qquad (31)$$
$$\lambda(b_6x_1 + b_7x_2 - b_8) = 0,\ u_3x_1 = 0, u_4x_2 = 0,$$
$$b_1x_1 + x_2 \le b_2,\ b_3x_1 + b_4x_2 \le b_5, b_6x_1 + b_7x_2 = b_8,$$
$$x_1, x_2 \ge 0, u_1 \ge 0, u_2 \ge 0, u_3 \ge 0, u_4 \ge 0$$

The problem (28) can be separated into two problems. The first one is:

$$\min\ 2x_1^2 + 2x_2^2 - 2x_1x_2 - 5x_1 + x_2$$
$$\text{subject to: } x_1 + x_2 \le 4,\ 2x_1 - x_2 \le 4, \qquad (32)$$
$$4x_1 - 8x_2 = 1.5,$$
$$x_1, x_2 \ge 0.$$

The second problem is:

$$\min\ 3x_1^2 + 2x_2^2 - 2x_1x_2 - 3x_1 + 2x_2$$
$$\text{subject to: } 2x_1 + x_2 \le 2,\ 3x_1 - 0.5x_2 \le 3, \qquad (33)$$
$$5x_1 - 7x_2 = 1,$$
$$x_1, x_2 \ge 0.$$

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Table 2 shows the results to problem 1, while figure 8 shows the convergence of problem 1 solution.

Table 2: The results to problem 1.

	(x_1, x_2)	$f(x)$	Time of computation
1st problem	$(1.5,0.5625)$	-3.4922	-
2nd problem	$(0.3268,0.0906)$	-0.5217	-
CGA	(0.7614,0.2554)	-1.7124	10.34E-002
CPSO	$(0.7754,0.2002)$	-2.7046	9.41E-002
CFA	$(1.4998,0.5624)$	-3.4922	8.95E-002
The stability set of first kind	$\left(\frac{2a_2+a_3}{2-4a_1}, \frac{a_2+a_1a_3}{2-4a_1}\right), \left(\frac{b_5-b_2b_4}{b_3-b_1b_4}, \frac{b_2b_3-b_1b_5}{b_3-b_1b_4}\right), \left(-\frac{a_2}{2a_1}, 0\right), \left(\frac{b_2}{b_1}, 0\right),$ $\left(\frac{b_4b_8-b_5b_7}{b_4b_6-b_3b_7}, \frac{b_5b_6-b_3b_8}{b_4b_6-b_3b_7}\right), \left(\frac{b_8-b_2b_7}{b_6-b_1b_7}, \frac{b_2b_6-b_1b_8}{b_6-b_1b_7}\right), \left(\frac{b_8}{b_6}, 0\right), \left(\frac{b_5}{b_3}, 0\right),$ $\left(\frac{2b_2(1+2b_1)-a_2+a_3b_1}{2a_1+4b_1(b_1+1)}, \frac{a_2b_1-a_3b_1^2+2b_2(a_1+b_1)}{2a_1+4b_1(b_1+1)}\right),$ $\left(\frac{a_3b_6b_7-a_2b_7^2+2b_8(2b_6+b_7)}{4b_6^2+4b_6b_7+2a_1b_7^2}, \frac{-a_3b_6^2+a_2b_6b_7+2b_8(b_6+a_1b_7)}{4b_6^2+4b_6b_7+2a_1b_7^2}\right),$ $\left(\frac{a_3b_3b_4-a_2b_4^2+2b_5(2b_3+b_4)}{4b_3^2+4b_3b_4+2a_1b_4^2}, \frac{a_2b_3b_4-a_3b_3^2+2b_5(b_3+a_1b_4)}{4b_3^2+4b_3b_4+2a_1b_4^2}\right).$		
Newton method	$x_1 = [0.28289, 9.8]$, $x_2 = [-5.4, 5.8]$.		

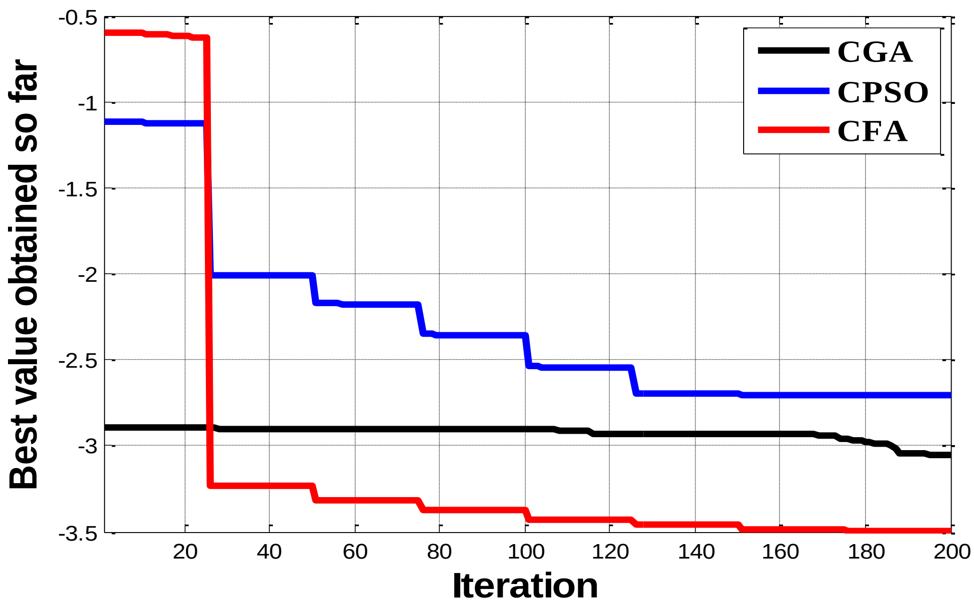

Fig 8: The convergence of problem 1.

5.2 Problem 2

The formulation of this problem is as follows [35]:

$$\min[-10,-6]x_1 + [2,3]x_2 + [4,10]x_1^2 + [-1,1]x_1x_2 + [10,20]x_2^2$$
$$\text{subject to:}[1,2]x_1 + 3x_2 \leq [1,10], \quad (34)$$

$$[-2,8]x_1 + [4,6]x_2 \leq [4,6],$$
$$x_1 \geq 0, x_2 \geq 0$$

All intervals are replaced by an additional parameter, the problem becomes:

$$\begin{gathered}\min a_1x_1 + a_2x_2 + a_3x_1^2 + a_4x_1x_2 + a_5x_2^2\\ \text{subject to:} b_1x_1 + 3x_2 \leq b_2,\\ b_3x_1 + b_4x_2 \leq b_5,\\ x_1, x_2 \geq 0\end{gathered} \tag{35}$$

where $a_1 = [-10, -6], a_2 = [2,3], a_3 = [4,10], a_4 = [-1,1], a_5 = [10,20]$, $b_1 = [1,2]$, $b_2 = [1,10], b_3 = [-2,8], b_4 = [4,6]$ and $b_5 = [4,6]$.

The dual form of problem (35) is:

$$\begin{gathered}\max a_1x_1 + a_2x_2 + a_3x_1^2 + a_4x_1x_2 + a_5x_2^2 + u_1(b_1x_1 + 3x_2 - b_2)\\ + u_2(b_3x_1 + b_4x_2 - b_5) - u_3(x_1) - u_4(x_2)\\ u_1 \geq 0, u_2 \geq 0, u_3 \geq 0, u_4 \geq 0,\end{gathered} \tag{36}$$

The KKTCs of problem (36) is:

$$\begin{gathered}a_1 + 2a_3x_1 + a_4x_2 + u_1b_1 + u_2b_3 - u_3 = 0,\\ a_2 + a_4x_1 + 2\,a_5x_2 + 3u_1 + u_2b_4 - u_4 = 0,\\ u_1(b_1x_1 + 3x_2 - b_2) = 0, u_2(b_3x_1 + b_4x_2 - b_5) = 0,\\ u_3(x_1) = 0, u_4(x_2) = 0,\ b_1x_1 + 3x_2 \leq b_2, b_3x_1 + b_4x_2 \leq b_5\\ x_1, x_2 \geq 0,\ u_1, u_2, u_3, u_4 \geq 0,\end{gathered} \tag{37}$$

The problem (34) can be separated into two problems. The first one is:

$$\begin{gathered}\min -10\,x_1 + 2x_2 + 4\,x_1^2 - x_1x_2 + 10\,x_2^2\\ \text{subject to: } x_1 + 3x_2 \leq 10\,,\\ -2\,x_1 + 4\,x_2 \leq 6,\\ x_1 \geq 0, x_2 \geq 0\end{gathered} \tag{38}$$

The second problem is:

$$\begin{gathered}\min -6\,x_1 + 3\,x_2 + 10\,x_1^2 + x_1x_2 + 20\,x_2^2\\ \text{subject to: } 2\,x_1 + 3x_2 \leq 1,\\ 8\,x_1 + 6\,x_2 \leq 4,\\ x_1 \geq 0, x_2 \geq 0\end{gathered} \tag{39}$$

Table 3 shows the results of problem 2, while figure 9 shows the convergence of problem 2 solution.

Table 3: The results of problem 2

	(x_1,x_2)	$f(x)$	Time of computation
1st problem	$(1.25,0)$	-6.25	-
2nd problem	$(0.3,0)$	-0.9	-
CGA	$(1.2504,0)$	-6.25	9.6E-002
CPSO	$(1.2504,0)$	-6.25	8.91E-002
CFA	$(1.2500,0)$	-6.25	7.65E-002
The stability set of first kind	$\left(-\frac{a_1}{2a_3},0\right),\left(\frac{b_2}{b_1},0\right),\left(-\frac{(a_2a_4-2a_1a_5)}{a_4^2-4a_3a_5},-\frac{0.5(a_2a_3-0.5a_1a_4)}{-0.25a_4^2+a_3a_5}\right).$		
Newton method	$x_1=[0.14625,10], x_2=[-\infty,\infty].$		

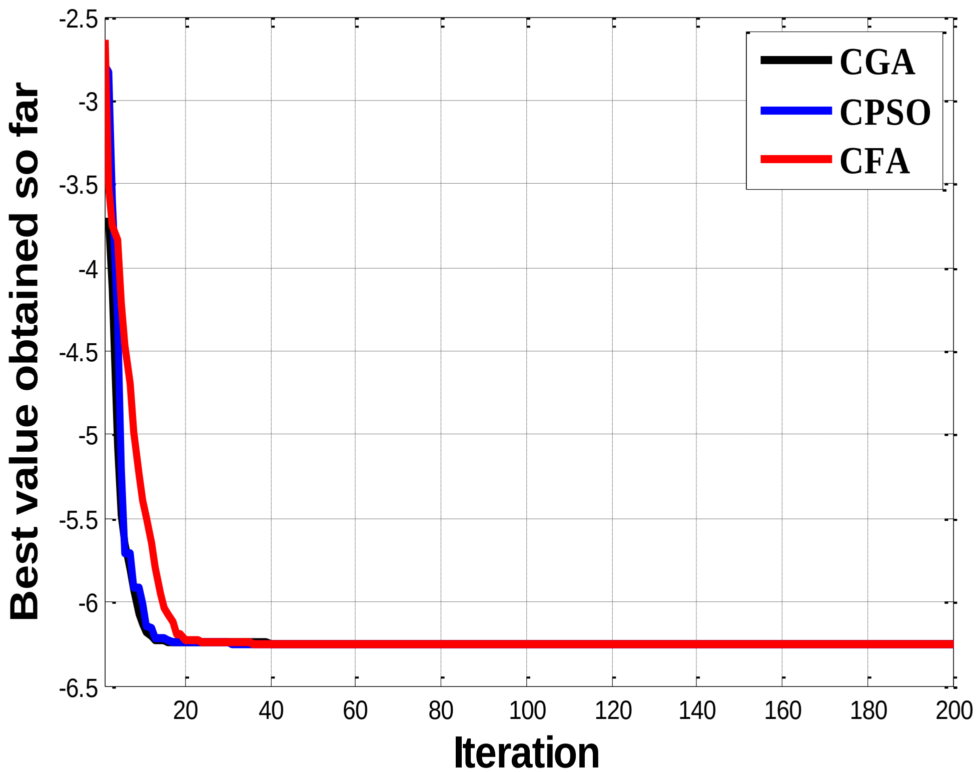

Fig 9: The convergence of problem 2

From tables 2-3 and the convergence solution figures 8-9, we can see that, in problem (1), problem (2), CGA, CPSO and CFA obtained the same as the solution of previous methods. While in problem (3) only the obtained solution by CFA is the same solution that obtained by the previous methods. In addition, the hybrid algorithms have a rapid convergence to the optimal solution as the CPU time is very small compared to the numerical methods that need high effort of computation to solve this type of problem. Finally, the numerical approach gives the solution as a general formula in the additional variables. In order to evaluate the suggested strategy with regard to the solution quality, comparative research has been done in this part. First, the quality of the solution is a problem for numerical approaches since they tend to fall into local optima, rely on the existence of derivatives, and are poorly resistant to discontinuous, enormous multimodal, and noisy search spaces. Therefore, by integrating EAs with CT, the suggested approach has been employed to get around these issues and improve the quality of the solutions. Second, the outcomes of the simulation demonstrated the superiority of the suggested strategy for solving INPP. To deliver a globally optimal solution, our method searches from a population of points. Additionally, because the proposed approach's methods are straightforward, complex INPP can be handled using it. Table 4 summarized the key distinctions between EAs, and the stability set of first kind. Table 4 shows that our approach offers good advantages for solving INPP, including quick computation times, a lack of sensitivity to initial conditions, and the ability to do global searches.

Table 4: Comparing between EAs and the stability set of first kind

Features	EAs	The stability set of first kind
High computational time	No	Yes
sensitive to the original conditions	No	Yes
Need introductory parameters	Yes	Yes
Capacity to search globally	Yes	No
Capacity to search locally	Yes	Yes

Contrarily, computational complexity refers to the amount of work required to solve a certain problem. The primary factor in the proposed algorithm's proposed algorithm's lower computational complexity is that our technique avoids the systematic search like in numerical methods. Additionally, our method yields an acceptable outcome. Additionally, our method may be utilized to achieve the ideal computational complexity, which is approximately order O(n2) and is unquestionably less than the computational complexity of the stability set of the first kind. Finally, the parallel execution capabilities of EAs can be leveraged, which can greatly reduce execution time.

5.3. Case study (Biogas Production Reactor)

Gaseous methane has been produced in a biogas production unite as illustrated in figure 10:

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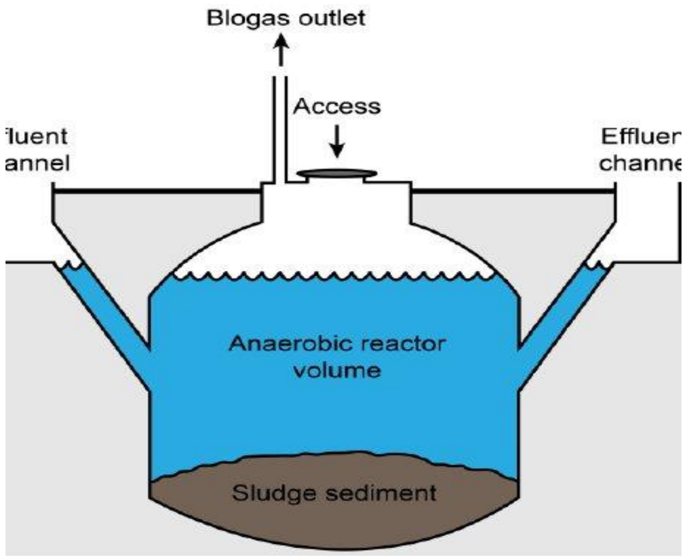

Fig 10: Biogas production reactor

The performance of biogas production unit (CH_4 production rate) as a function of operating parameter, retention time (t,s), inlet feed rate (kg/d), reaction temperature (T, C^o), pH. And water content W. Based on the assumption that biogas production is a linear function of the operation conditions formulate the maximum production as a linear optimization problem.

$$\max SNRA(S/_N) = [-187,-186] + [-1.4,-1.3]M + [1.9,2]T + [41,42]pH + [1.7,1.8]W + [1.5,1.6]t + [0.014,0.015]M^2 + [-0.02,-0.019]T^2 + [-3.2,-3.1]pH^2 + [-0.07,-0.06]W^2 + [-0.022,-0.021]t^2; \tag{40}$$

where $5 \leq pH \leq 7, 20 \leq t \leq 40, 40 \leq T \leq 60, 5 \leq W \leq 15, 40 \leq M \leq 60.$

$$S/_N = -10\log\left(\frac{1}{n}\sum_{k=1}^{n}\frac{1}{y_i^2}\right) \tag{41}$$

The signal-to-noise ratio (S/N) is obtained by Eq. (41), where n is the run number, and y is the response (m^3/d) of system [59].
The problem becomes after replacing all intervals by new parameters:

$$\max SNRA\left({}^{S}/_{N}\right) = a_1 + a_2M + a_3T + a_4pH + a_5W + a_6t + a_7M^2 + a_8T^2 + a_9pH^2 + a_{10}W^2 + a_{11}t^2$$
$$5 \le pH \le 7, 20 \le t \le 40, 40 \le T \le 60, 5 \le W \le 15, 40 \le M \le 60, \quad (42)$$
$$a_1 = [-187,-186], a_2 = [-1.4,-1.3], a_3 = [1.9,2], +a_4 = [41,42],$$
$$a_5 = [1.7,1.8], a_6 = [1.5,1.6], a_7 = [0.014,0.015], a_8 = [-0.02,-0.019],$$
$$a_9 = [-3.2,-3.1], a_{10} = [-0.07,-0.06], a_{11} = [-0.022,-0.021]$$

The dual form of problem (42) is:

$$\min a_1 + a_2M + a_3T + a_4pH + a_5W + a_6t + a_7M^2 + a_8T^2 + a_9pH^2 + a_{10}W^2 + a_{11}t^2 + u_1(5-pH) - u_2(7-pH) + u_3(20-t) - u_4(40-t) + u_5(40-T) - u_6(60-T) + u_7(5-W) - u_8(15-W) + u_9(40-M) - u_{10}(60-M). \quad (43)$$
$$u_1 \ge 0, u_2 \ge 0, u_3 \ge 0, u_4 \ge 0, u_5 \ge 0, u_6 \ge 0, u_7 \ge 0, u_8 \ge 0, u_9 \ge 0, u_{10} \ge 0.$$

The KKTCs of problem (43) is:

$$a_2 + 2a_7M - u_9 + u_{10} = 0,$$
$$a_3 + 2a_8T - u_5 + u_6 = 0,$$
$$a_4 + 2a_9pH - u_1 + u_2 = 0,$$
$$a_5 + 2a_{10}W - u_7 + u_8 = 0,$$
$$a_6 + 2a_{11}t - u_3 + u_4 = 0,$$
$$u_1(5-pH) = 0, u_2(7-pH) = 0, u_3(20-t) = 0,$$
$$u_4(40-t) = 0, u_5(40-T) = 0, u_6(60-T) = 0, \quad (44)$$
$$u_7(5-W) = 0, u_8(15-W) = 0, u_9(40-M) = 0,$$
$$u_{10}(60-M) = 0, 5 \le pH \le 7, 20 \le t \le 40,$$
$$40 \le T \le 60, 5 \le W \le 15, 40 \le M \le 60,$$
$$u_1 \ge 0, u_2 \ge 0, u_3 \ge 0, u_4 \ge 0, u_5 \ge 0, u_6 \ge 0,$$
$$u_7 \ge 0, u_8 \ge 0, u_9 \ge 0, u_{10} \ge 0.$$

The problem (40) can be separated into two problems. The first one is:

$$\max SNRA\left({}^{S}/_{N}\right) = -186 - 1.3M + 2T + 42pH + 1.8W + 1.6t + 0.015M^2 - 0.019T^2 - 3.1pH^2 - 0.06W^2 - 0.021t^2. \quad (45)$$
$$5 \le pH \le 7, 20 \le t \le 40, 40 \le T \le 60, 5 \le W \le 15, 40 \le M \le 60,$$

The second problem is:

$$\max SNRA\left({}^{S}/_{N}\right) = -187 - 1.4M + 1.9T + 41pH + 1.7W + 1.5t + 0.014M^2 - 0.02T^2 - 3.2pH^2 - 0.07W^2 - 0.022t^2. \quad (46)$$
$$5 \le pH \le 7, 20 \le t \le 40, 40 \le T \le 60, 5 \le W \le 15, 40 \le M \le 60$$

Table 5 shows the results of Gaseous methane problem, while figure 11 shows the convergence of Gaseous methane problem solution.

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Table 5: The results of Bigas production process

	(x_1, x_2)	$f(x)$	Time of computation
1st problem	$T = 60, M = 60, pH = 7, W = 15, t = 40$	27.6	
2nd problem	$T = 60, M = 60, pH = 7, W = 15, t = 40$	-12.35	
CGA	$T = 55.0126017006333,$ $M = 60,$ $pH = 6.75229002085684,$ $W = 15,$ $t = 36.4783272201342$	28.644113523673806	0.005
CPSO	$T = 50.8100120327578,$ $M = 59.6390405961163,$ $pH = 6.75084181196437,$ $W = 14.6427608734007,$ $t = 36.1293051153934$	28.40621169410944	0.015
CFA	$T = 50.4381469411430,$ $M = 59.9256323571470,$ $pH = 6.73594613050007,$ $W = 14.5781876462973,$ $t = 36.2078252333204$	28.522083984553890	0.019
The stability set of first kind	$T = \left(40 \vee 60 \vee -\frac{a_3}{2a_8}\right), M = \left(-\frac{a_2}{2a_7}\right), pH = \left(-\frac{a_4}{2a_9} \vee 5 \vee 7\right), W = -\frac{a_5}{2a_{10}} \vee 5, t = \left(-\frac{a_6}{2a_{11}} \vee 20 \vee 40\right)$		
Newton method	$T = [40,60], M = [40,60], \mathrm{pH} = [5,7], W = [5,15], t = [20,40]$.		

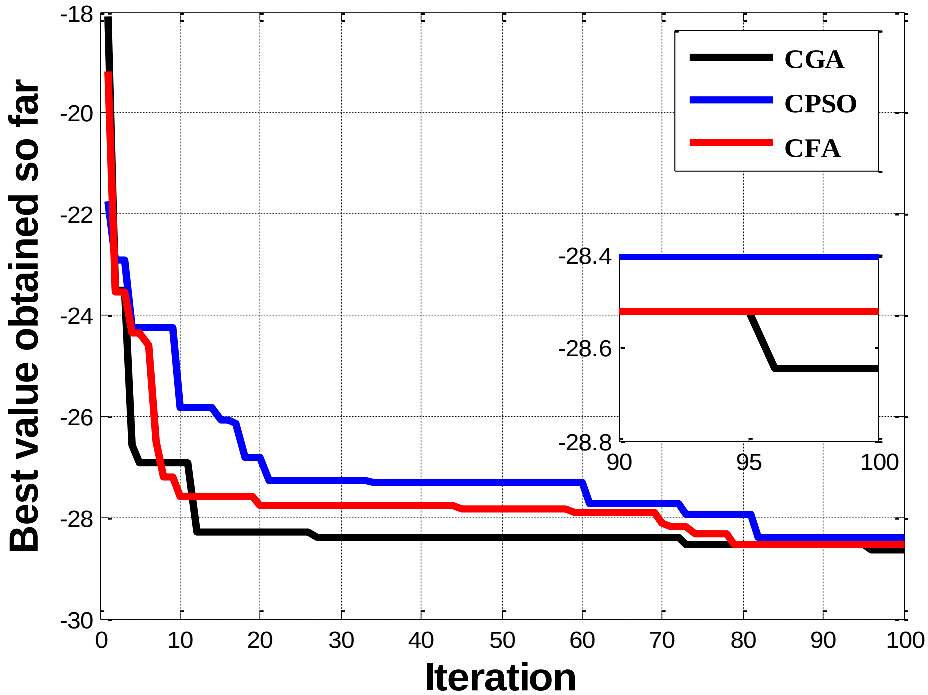

Fig 11: The convergence of Gaseous methane problem solution.

5.3.1. Process analysis

Taguchi method has been recommended to limit the influence of uncontrollable factors of biogas production processes, an orthogonal arrays L27 has been constructed. Experimental data of biogas unit has been collected from biogas production process. The operating parameters have been screened to estimate the most influencing parameters and interaction between each other. five input independent variables have been chosen to be, mass of cow dung ($M, Kg/day$), pH, T (C^o), amount of water W%, and reaction retention time t (day). System response has been found to be bio-gas flow rate (m^3/d). The selected variables have been converted to coded factors of M, W, T, pH and t as shown in table 6.

Table 6: Levels of Process parameters

L	Unit	Level		
		L_1	L_2	L_3
M	Kg/d	40	50	60
W	W%	5	10	15
T	C^o	40	50	60
pH	pH	5	6	7
t	day	20	30	40

Signal-to-noise ratio has been utilized in measurable amounts of qualitative characteristics according to the response is larger is better, Eq. (42).

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5.3.2. Optimization of process parameters based on $S/_{N}$ ratio

As illustrated in table 7 and figure 12, system response is expressed as, SNR for biogas volumetric flow rate. Table 7 and figure 12 show the results of the software's independent calculations of each element's impact at each level, followed by an analysis of each factor's relative importance based on the differences each factor caused.

Table 7: Response for $S/_{N}$ Ratios (Larger is better)

Level	M	T	pH	W	t
1	-0.3762	-0.5310	-4.7134	-2.4246	-3.1224
2	-0.8781	1.4117	2.2219	1.2636	1.5525
3	1.5757	-0.5593	2.8129	1.4823	1.8911
Delta	2.4538	1.9710	7.5263	3.9070	5.0135
Rank	4	5	1	3	2

Based on the S/N ratio, the process parameters were optimized. Volumetric biogas flow rate is larger the larger the S/N ratio and vice versa. From table 7, it can be observed that, the S/N ratio is largest 1.572.81, 1,48, 2.81 and 1.89 at level '3' of factor of M, W, pH and t respectively and 1.41 at level (2) for T. This observation has been confirmed to that displayed in figure 12.

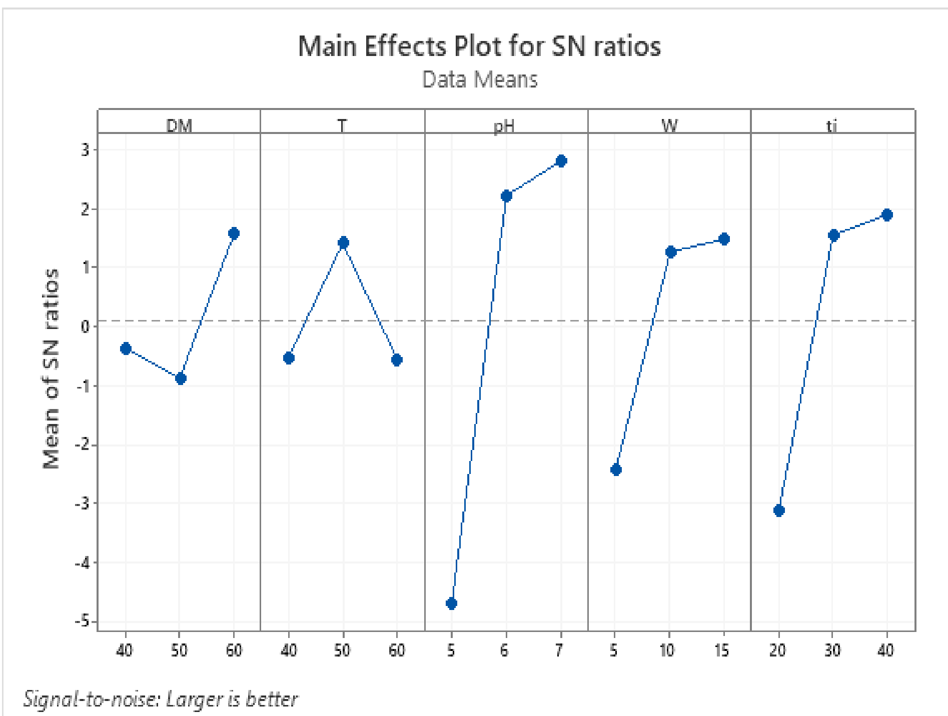

Fig 12: Signal to noise ratios for biogas production process

Figure 12 describes the relation between mean of the S/N ratios and the input parameters M, W, T, pH and t) with larger is better method in Taguchi Design analysis for biogas production process. It has been proved that biogas volumetric flow rate is maximum at level 3 of M, W, pH and t and at level (2) for T.

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5.3.3. Analysis of Variance (ANOVA)

ANOVA is a reliable tool for evaluating a matched model's quality. ANOVA table for biogas production process are presented in table 8. From this table the mathematical model is congruent with the experimental results with correlation coefficients of R^2 of 96.41 and R^2-(adj) of 92.22. A factor of p-value less than 0.05 is achieved for all the examined factors. (M, W, T, pH and t). High values of R^2 and adj-R^2 support the model's ability to produce a believable response estimate.

Table 8: Analysis of variance for biogas production process

Source	DF	Seq SS	Contribution	Adj SS	Adj MS	F-Value	P-Value
M	2	30.24	4.84	30.248	15.124	8.16	0.004
W	2	22.98	4.743.67	22.978	11.489	6.20	0.010
T	2	315.28	50.35	315.279	157.639	85.03	0.000
pH	2	86.75	13.85	86.746	43.373	23. 40	0.000
t	2	141.31	22.57	141.314	70.657	38.11	0.000
Error	16	29.66	-	29.490	1.874	-	-
Total	26	626.23	100%	626.226	-	-	-

The mathematical model is presented as a hierarchical model which in turn relates process operating parameters (M, W, T, pH and t) to describe the significance of each parameter in the response of biogas volumetric flow rate, Eq. (40). Model regression correlation factors are found to be R^2 of 95.26 and R^2-(adj) of 92.3.

6. CONCLUSION

In this paper, INPP was solved; where all the intervals in the INPP were replaced by an additional variable and the new form is called MIPP. In addition, three hybrid EAs which are CGA, CPSO and CFA to solve MIPP. The hybrid algorithms integrate; three of EAs with CT. In addition, The KKT conditions for MIPP which were presented. The stability set of first kind is derived for INPP. Also, a new a stopping criterion is presented for the hybrid algorithms. The outcomes have proven the superiority of the suggested strategy to address INPP. The suggested optimization algorithm has the following advantages, which are easily observed:

1. The proposed approach has the best performance, accelerates the optimum seeking operation, and finds the global optimal solution.
2. The recommended method can be used to resolve problems that are both convex and nonconvex.
3. The hybrid algorithms have a rapid convergence to the optimal solution as the CPU time is very small compared to the stability set of first kind.
4. We define the stability set of first kind when the parameters in the objective function and constrains.
5. Our proposed approach establishes the relationship between optimal solution of the interval programming problem and the change of the interval coefficients, which consider as important relation from the point of view to the decision maker. Also,

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the proposed approach presents this relationship between changing variables and the stability of the optimal solution.

6. The proposed approach procedures are straightforward, making it possible to apply them to complex problems.
7. CGA, CPSO, and CFA combine the potent local searching powers of CT with the global searching capabilities of GA, FA, and PSO.
8. Biogas production is one of the easiest biofuels conversions from animal/human manure.
9. Biogas may be easily used to run gas engines and generate electrical power.
10. The optimal conditions of producing bigas from a continuous digestor is pH 6.8-7, T= 40 C^o, 50 kg of cow dung, 30day and 10% water content.
11. 50 kg of cow dung produces 2m3/d of biogas.

The results were promising, so this approach could be applicable for many real-life problems.

Data Availability

All data used to support the findings of this study are included within the article.

Declaration of interest

The authors declare that this article content has no conflict of interest.

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